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Tests of the final-state radiation model at DAFNE near $\pi^+\pi^-$ threshold

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Abstract. Effects due to the non-pointlike behaviour of pions in the process $e^+e^- \to \pi^+\pi^-\gamma$ can arise for hard photons in the final state. By means of a Monte Carlo event generator, which also includes the contribution of the direct decay $\phi \to \pi^+\pi^-\gamma$, we estimate these effects in the framework of the resonance perturbation theory. We consider angular cuts used in the KLOE analysis of the pion form factor at threshold. A method to reveal the effects of the non-pointlike behaviour of pions in a model-independent way is proposed.

PACS. 13.25.Jx Decays of other mesons – 12.39.Fe Chiral Lagrangians – 13.40.Gp Electromagnetic form factors

Final-state radiation (FSR) is the main irreducible background in radiative return measurements of the hadronic cross-section [1] which is important for the anomalous magnetic moment of the muon [2]. Besides being of interest as an important background source, this process could be of interest in itself, because a detailed experimental study of FSR allows us to get information about the pion-photon interaction at low energies. Differently from ISR, whose accuracy is limited by the numerical precision on the evaluation of high-order QED diagrams (see, for example, [3,4] and discussion there), the FSR evaluation relies on specific models for the coupling of hadrons to photons. Usually, the FSR amplitude in the process $e^+e^- \to \pi^+\pi^-\gamma$ is evaluated in the scalar QED (sQED) model, where the pions are treated as point-like particles and the total FSR amplitude is multiplied by the pion form factor computed in the VMD model [4,5]. While this assumption is generally valid for relatively soft photons, it can fail for low values of the invariant mass of the hadronic system, *i.e.* when the intermediate hadrons are far off shell.

In general case the cross-section of the reaction

$$e^+(p_1) + e^-(p_2) \to \pi^+(p_+)\pi^-(p_-)\gamma(k)$$

with the photon emitted in the final state, can be written as

$$\mathrm{d}\sigma = \frac{1}{2s(2\pi)^5}\int \delta^4(P - p_+ - p_- - k)\frac{\mathrm{d}^3p_+\mathrm{d}^3p_-\mathrm{d}^3k}{8E_+E_-\omega}|M|^2, \tag{1}$$

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where $P = p_1 + p_2$, $s = P^2$,

$$M = \frac{e}{s}M^{\mu\nu}\bar{u}(-p_1)\gamma_\mu u(p_2)\epsilon^\star_\nu \tag{2}$$

and the tensor $M^{\mu\nu}$ describes the process

$$\gamma^*(P) \to \pi^+(p_+)\pi^-(p_-)\gamma(k).$$

Based on charge conjugation symmetry, photon crossing symmetry and gauge invariance, $M^{\mu\nu}$ can be expressed by three gauge invariant tensors

$$\begin{aligned}
M^{\mu\nu}(P,k,l) &\equiv -ie^2M_F^{\mu\nu}(P,k,l) = \\
&-ie^2(\tau_1^{\mu\nu}f_1 + \tau_2^{\mu\nu}f_2 + \tau_3^{\mu\nu}f_3), \quad l = p_+ - p_-, \\
\tau_1^{\mu\nu} &= k^\mu P^\nu - g^{\mu\nu}k\cdot P, \\
\tau_2^{\mu\nu} &= k\cdot l(l^\mu P^\nu - g^{\mu\nu}k\cdot l) + l^\nu(k^\mu k\cdot l - l^\mu k\cdot P), \\
\tau_3^{\mu\nu} &= P^2(g^{\mu\nu}k\cdot l - k^\mu l^\nu) + P^\mu(l^\nu k\cdot P - P^\nu k\cdot l).
\end{aligned}$$

While the last decomposition is general and does not depend on the FSR mechanism, the exact value of the scalar functions f_i (form factors), each depending in terms of three independent variables, is determined by the specific FSR models. In the paper [6] the prediction for f_i in the framework of the resonance perturbation theory (RPT) was considered. We would like to remind that RPT is a model based on chiral perturbation theory (χPT) with the explicit inclusion of the vector and axial-vector mesons, $\rho_0(770)$ and $a_1(1260)$ [7]. Whereas χPT gives correct predictions for the pion form factor at very low energy, RPT is the appropriate framework to describe the pion form factor at intermediate energies ($E \sim m_\rho$) [7].

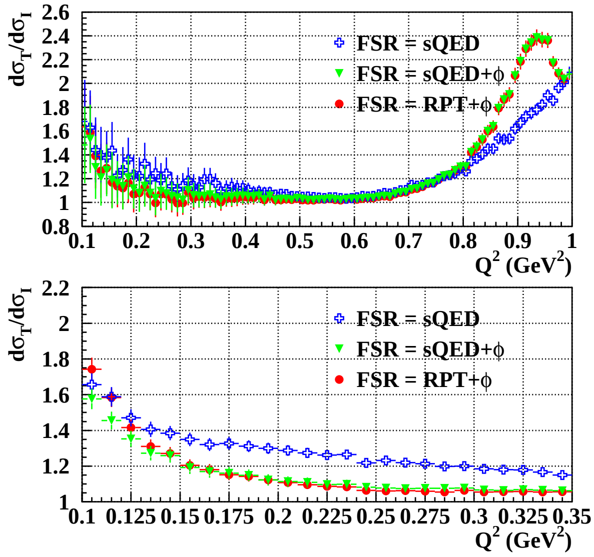

Fig. 1. The ratio $\mathrm{d}\sigma_T/\mathrm{d}\sigma_I$ as a function of the invariant mass of the two pions, in the region $50^\circ \leq \theta_\gamma \leq 130^\circ$, $50^\circ \leq \theta_\pi \leq 130^\circ$, for different models of FSR at $s = m_\phi^2$.

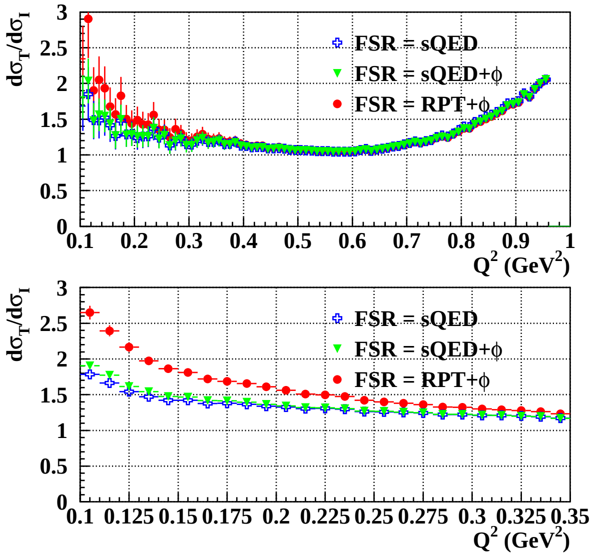

Fig. 2. The ratio $\mathrm{d}\sigma_T/\mathrm{d}\sigma_I$ as a function of the invariant mass of the two pions, in the region $50^\circ \leq \theta_\gamma \leq 130^\circ$, $50^\circ \leq \theta_\pi \leq 130^\circ$, for different models of FSR at $s = 1\,\mathrm{GeV}^2$.

At $s = m_\phi^2$, DAFNE energies, an additional complication arises: the presence of the direct decay $\phi \to \pi^+\pi^-\gamma$. Its contribution should be included in a realistic Monte Carlo generator (MC).

It is convenient to write down the differential cross-section for the reaction $e^+e^- \to \pi^+\pi^-\gamma$, where the FSR amplitude (M_{FSR}) receives contributions both from RPT (M_{RPT}) and the $\phi \to \pi^+\pi^-\gamma$ decay (M_ϕ), as

$$\begin{aligned} \mathrm{d}\sigma_T &\sim |M_{ISR} + M_{FSR}|^2 = \mathrm{d}\sigma_I + \mathrm{d}\sigma_F + \mathrm{d}\sigma_{IF}, \\ \mathrm{d}\sigma_I &\sim |M_{ISR}|^2, \ \mathrm{d}\sigma_{IF} \sim 2\,\mathrm{Re}\{M_{ISR} \cdot (M_{RPT} + M_\phi)^*\}, \\ \mathrm{d}\sigma_F &\sim |M_{RPT}|^2 + |M_\phi|^2 + 2\,\mathrm{Re}\{M_{RPT} \cdot M_\phi^*\}. \end{aligned} \quad (3)$$

The ϕ direct decay is described by Achasov four-quark model with parameters extracted from the fit to $\phi \to \pi^0\pi^0\gamma$ [8]. The interference term $\mathrm{d}\sigma_{IF}$ is equal to zero for symmetric cuts on the polar angle of the pions. We consider only the case of destructive interference between the two amplitudes ($\mathrm{Re}(M_{RPT} \cdot M_\phi^*) < 0$). Published data from the KLOE experiment [1] are in favour of this assumption, which we will use in the following.

In figs. 1 and 2 we show the values of $\mathrm{d}\sigma_T/\mathrm{d}\sigma_I$ for the angular cuts of the KLOE large-angle analysis $50^\circ \leq \theta_\gamma \leq 130^\circ$, $50^\circ \leq \theta_\pi \leq 130^\circ$, with and without contributions from RPT and ϕ direct decay, for a hard-photon radiation with energies $E_\gamma > 20\,\mathrm{MeV}$ for $s = m_\phi^2$ and $s = 1\,\mathrm{GeV}^2$.

For $s = m_\phi^2$ three distinctive features can be noted: 1) the peak at about $1\,\mathrm{GeV}^2$ corresponds to the f_0 intermediate state for the $\phi \to \pi\pi\gamma$ amplitude; 2) the presence of RPT terms in the FSR is relevant at low Q^2, where they give an additional contribution up to 40% to the ratio $\mathrm{d}\sigma_{RPT+\phi}/\mathrm{d}\sigma_{sQED+\phi}$; 3) the destructive interference with the ϕ direct decay amplitude reduces $\mathrm{d}\sigma_F$ at low Q^2 (see fig. 1, down). Also the dependence of the cross-section on the FSR model is decreased due to the destructive interference.

In the case of $s = 1\,\mathrm{GeV}^2$ the ϕ resonant contribution is suppressed ($\mathrm{d}\sigma_T$ with and without the ϕ direct decay almost coincide), see fig. 2. Therefore the main contribution beyond sQED to the FSR cross-section comes from RPT. As expected the presence of RPT gives relevant effects at the low-Q^2 region, while the presence of a bump at high Q^2 is due to the ϕ direct decay. For a more detailed discussion and results for forward-backward asymmetry see [9,10].

Contributions to FSR beyond sQED, as in the case of RPT, can lead to sizeable effects on the cross-section and asymmetry at threshold, as shown in figs. 1 and 2. A precise measurement of the pion form factor in this region needs to control the FSR cross-section at an accuracy better than 1% [11]. This looks like a rather difficult task, if one thinks that effects beyond sQED, as well as the contribution from $\phi \to \pi^+\pi^-\gamma$, are model dependent. In ref. [10] a method for a model-independent analysis of the FSR contribution beyond sQED was proposed. The main idea of this method is the following: we propose to consider a quantity that can be related to experimental spectra and that has a very well-described behaviour in sQED. In our opinion, this quantity can be determined as

$$\begin{aligned} \Delta Y(Q^2) &= Y_{s_1}(Q^2) - Y_{s_2}(Q^2), \\ Y_s(Q^2) &= \frac{\left(\frac{\mathrm{d}\sigma_T}{\mathrm{d}Q^2}\right)_s - \left(\frac{\mathrm{d}\sigma_{sQED+\phi}}{\mathrm{d}Q^2}\right)_s}{H_s(Q^2)}, \\ &= |F_\pi(Q^2)|^2 + \Delta F_s(Q^2), \end{aligned} \quad (4)$$

where s_1 and s_2 are two different c.m. energy of e^+e^-, for KLOE setup $s_1 = 1\,\mathrm{GeV}^2$ and $s_2 = m_\phi^2$, $\frac{\mathrm{d}\sigma_T}{\mathrm{d}Q^2}$ is the differential cross-section of the process $e^+e^- \to \pi^+\pi^-\gamma$ (photon

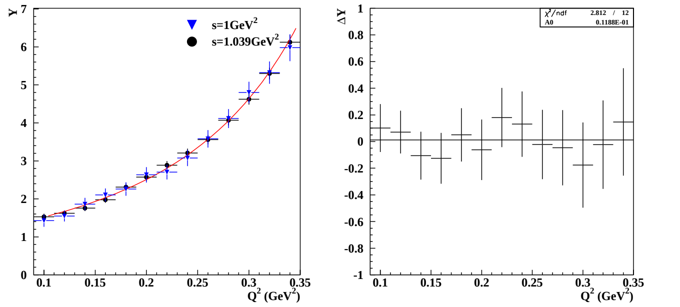

Fig. 3. Left: $Y_s(Q^2)$ at $s = 1\,\mathrm{GeV}^2$ (triangles), and at $s = m_\phi^2$ (circles), when FSR includes only sQED and ϕ contribution. The pion form factor $|F_\pi(Q^2)|^2$ is shown by the solid line. Right: the difference $\Delta Y(Q^2)$.

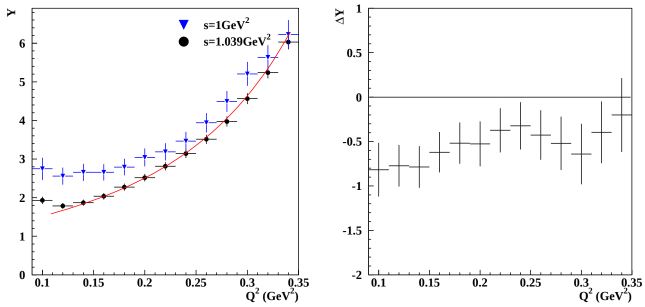

Fig. 4. Left: $Y_s(Q^2)$ at $s = 1\,\mathrm{GeV}^2$ (triangles), and at $s = m_\phi^2$ (circles), when FSR includes RPT and ϕ contribution. The pion form factor $|F_\pi(Q^2)|^2$ is shown by the solid line. Right: the difference $\Delta Y(Q^2)$.

can be radiated both from final state and by leptons) and $\frac{\mathrm{d}\sigma_{sQED+\phi}}{\mathrm{d}Q^2}$ is the differential cross-section only for FSR in the framework of sQED and due to ϕ direct decay. In the framework of sQED the value of $Y(Q^2)$ coincides with the square of the pion form factor and does not depend on the initial energy. In other words for sQED $\Delta Y(Q^2) = 0$. That means that any deviation from zero will be in favour of some contribution beyond sQED. In experimental conditions this theoretical idea can be affected by the statistical uncertainty. To check if the number of events collected by KLOE ($200\,\mathrm{pb}^{-1}$ at $1\,\mathrm{GeV}^2$ and $2.5\,\mathrm{fb}^{-1}$ at m_ϕ^2) is enough to show a possible deviation from sQED, we applied the results of our MC when FSR is calculated by sQED or RPT. Figure 3 shows the quantity $Y_s(Q^2)$ at $s_1 = 1\,\mathrm{GeV}^2$ and at $s_2 = m_\phi^2$ and the value $\Delta Y(Q^2)$, when FSR is described by sQED. As expected, each of the quantities Y_{s_1} and Y_{s_2} coincides with the square of the pion form factor $|F_\pi(Q^2)|^2$, shown by solid line. The value of ΔY is consistent with zero. A combined fit of Y_{s_1} and Y_{s_2} to the pion form factor is also possible:

$$F_\pi(Q^2) \simeq 1 + p_1 * Q^2 + p_2 * q^4. \tag{5}$$

It gives the following values: $p_1 = 1.4 \pm 0.186\,\mathrm{GeV}^{-2}$, $p_2 = 8.8 \pm 0.73\,\mathrm{GeV}^{-4}$, $\chi^2/\nu = 0.25$.

A different situation appears if the FSR emission from pions is modeled by RPT. In this case, as shown in fig. 4, the difference $\Delta Y(Q^2) \neq 0$ and the quantities $Y_s(Q^2)$ cannot be anymore identified with $|F_\pi(Q^2)|^2$. A combined fit of Y_{s_1} and Y_{s_2} is no longer possible.

At the end we would like to remind the main points of this work:

- Test of FSR at threshold in the process $e^+e^- \to \pi^+\pi^-\gamma$ is an important issue to get information about pion-photon interaction when the intermediate hadrons are far off shell.
- At $s = m_\phi^2$ an additional complication arises: the presence of the direct decay $\phi \to \pi^+\pi^-\gamma$ whose amplitude and relative phase can be described according to some model.
- We show that the low-Q^2 region is sensitive to the inclusion of additional terms beyond sQED in the FSR cross-section.
- A method to study FSR by model-independent way based on the experimental spectra is proposed.

Work is in progress to include the $f_0 + \sigma$ parametrization for the ϕ direct decay (instead of only the f_0 one, as it has been done in this paper) and consider the $\phi \to \rho\pi \to \pi\pi\gamma$ decay.

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